Conflict-Driven Clause Learning

Sasha Fedchin[1]

[1]Department of Computer Science
Tufts University

Motivation: why DPLL is not enough

Looking for satisfying interpretation of:

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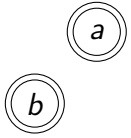

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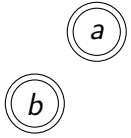

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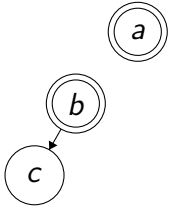

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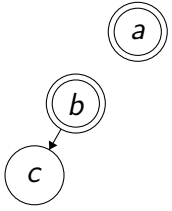

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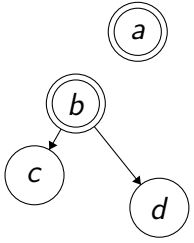

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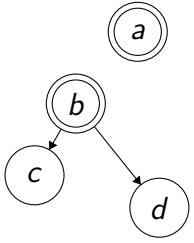

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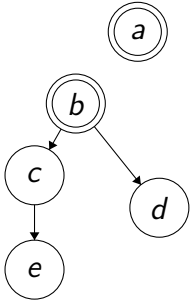

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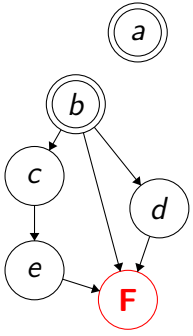

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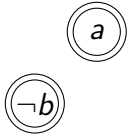

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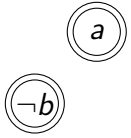

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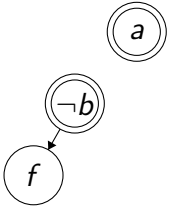

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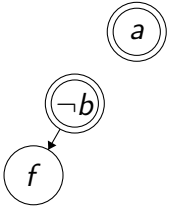

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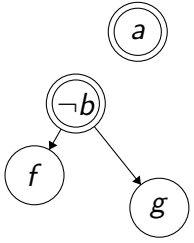

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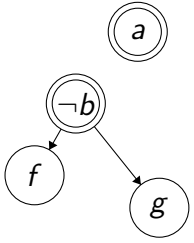

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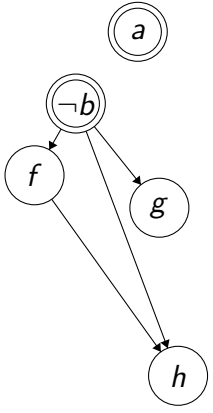

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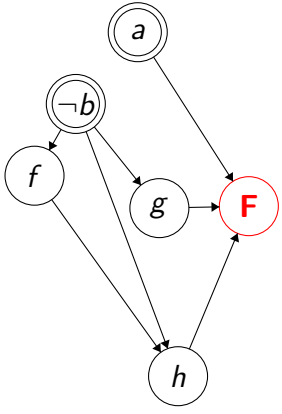

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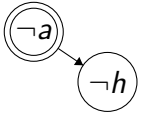

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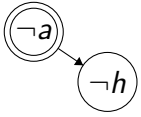

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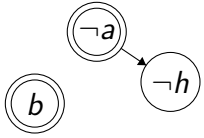

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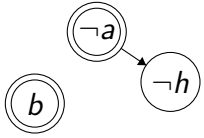

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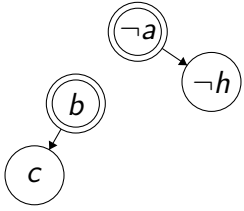

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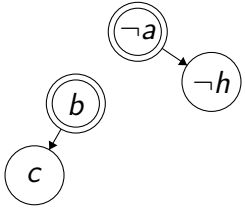

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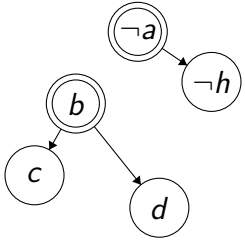

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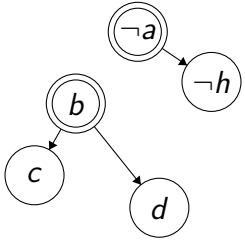

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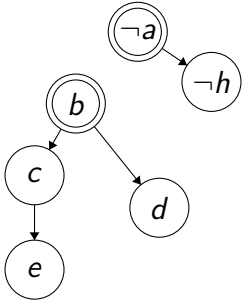

Motivation: why DPLL is not enough

Looking for satisfying interpretation of:

$$\wedge\,(a \vee \neg h)$$
$$\wedge\,(\neg b \vee c)$$
$$\wedge\,(\neg b \vee d)$$
$$\wedge\,(\neg c \vee e)$$
$$\wedge\,\boxed{(\neg e \vee \neg d \vee \neg b)}$$
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$$\wedge\,(b \vee g)$$
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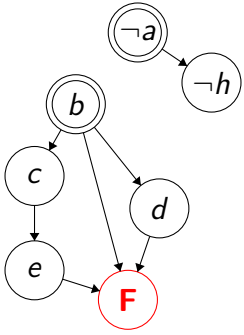
$$\wedge\,(\neg h \vee \neg g \vee \neg a)$$

Implication Graph:



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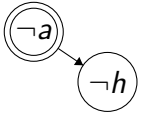
Implication Graph:



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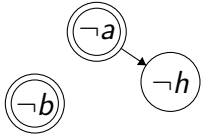
Implication Graph:



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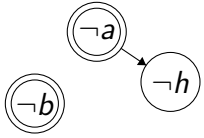

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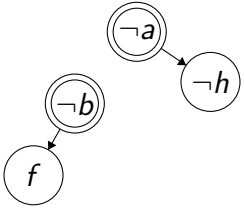
Implication Graph:



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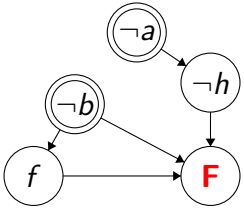
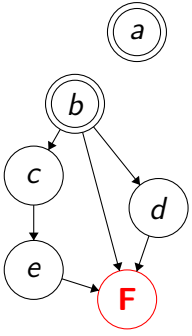
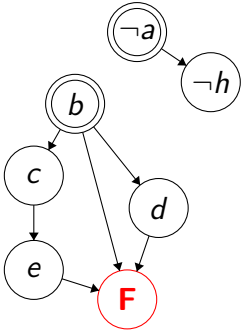
Implication Graph:



Conflict Driven Clause Learning - Motivation

- We have encountered two conflicts that had similar implication graphs:





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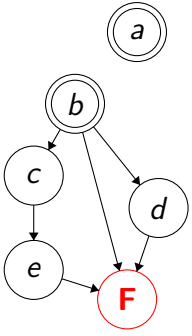
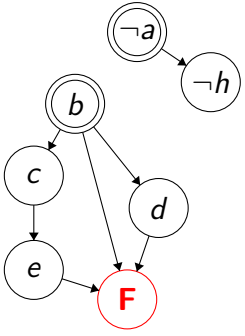
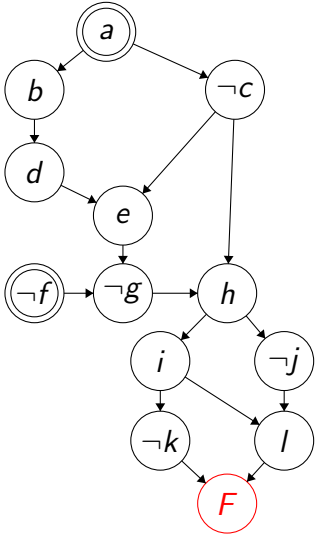


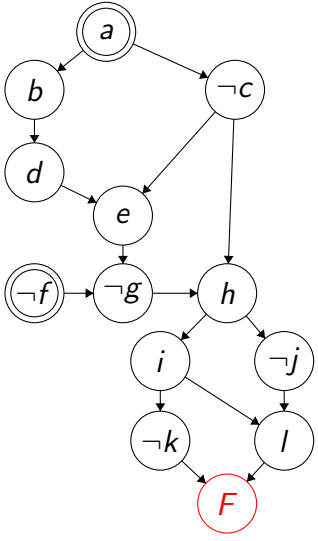


- We can learn the $\neg b$ clause from the conflict on the left and then avoid deriving it for the second time. This is conflict-driven clause learning.

Learning Clauses from Conflicts

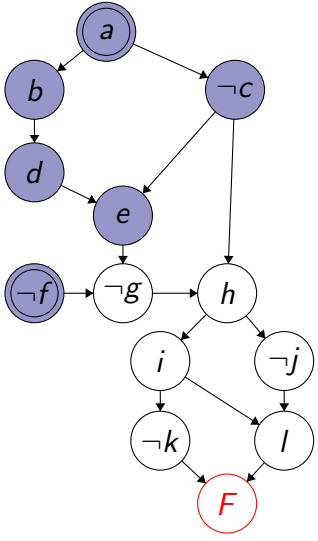


Learning Clauses from Conflicts



To learn a clause:

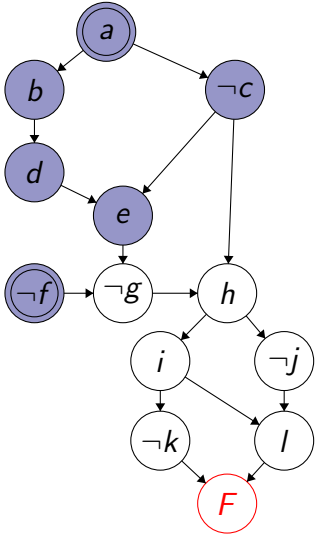
Learning Clauses from Conflicts



To learn a clause:

- Cut the graph in two sets A and B.

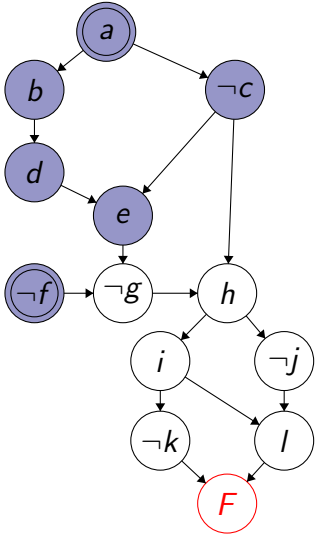
Learning Clauses from Conflicts



To learn a clause:

- Cut the graph in two sets A and B.
- Decision vertices a and $\neg f$ must belong to A.

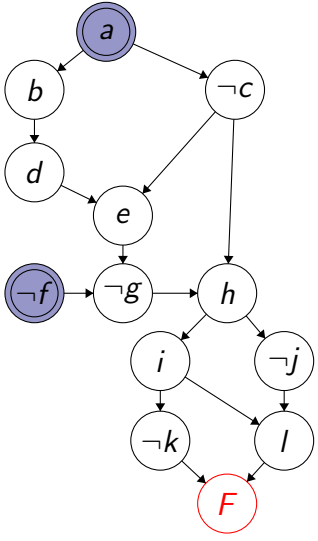
Learning Clauses from Conflicts

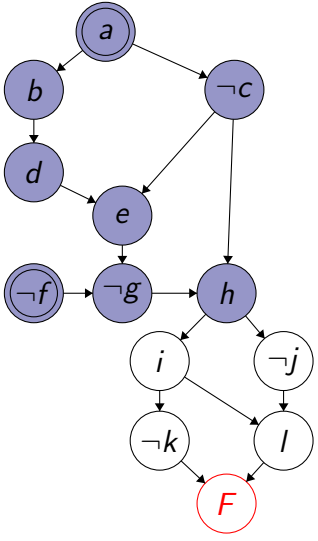


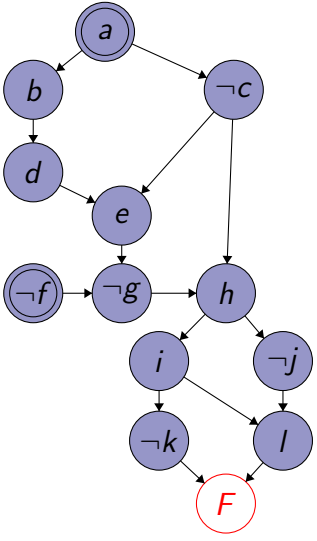
To learn a clause:

- Cut the graph in two sets A and B.
- Decision vertices a and $\neg f$ must belong to A.
- Vertices in A that have edges into B form the literals of a new clause $(f \vee \neg e \vee c)$.

Multiple cuts are possible







CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
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&\wedge (\neg b \vee c) \\
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&\wedge (\neg c \vee e) \\
&\wedge (\neg e \vee \neg d \vee \neg b) \\
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Implication Graph:

CDCL: Motivating Example Revisited

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CDCL: Motivating Example Revisited

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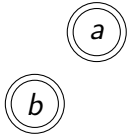
Implication Graph:



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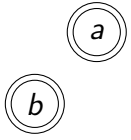

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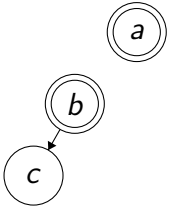

CDCL: Motivating Example Revisited

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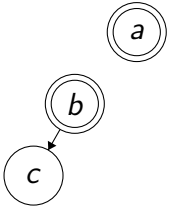

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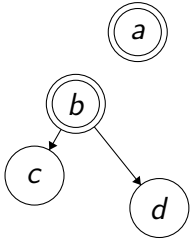

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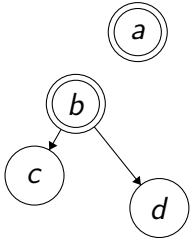

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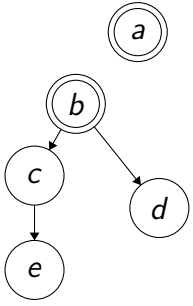

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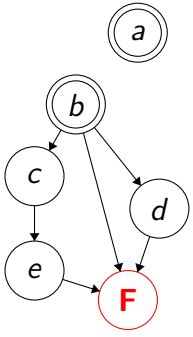

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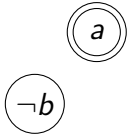

CDCL: Motivating Example Revisited

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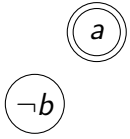

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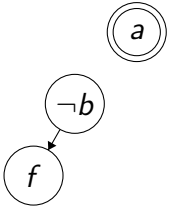

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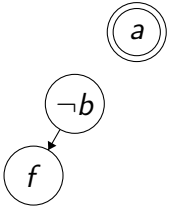
Implication Graph:



CDCL: Motivating Example Revisited

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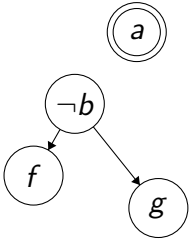

CDCL: Motivating Example Revisited

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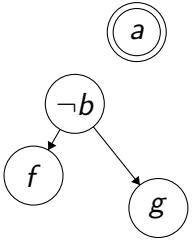
$$\wedge\ (\neg b \vee d)$$
$$\wedge\ (\neg c \vee e)$$
$$\wedge\ (\neg e \vee \neg d \vee \neg b)$$
$$\wedge\ (b \vee f)$$
$$\wedge\ (b \vee g)$$
$$\wedge\ (\neg f \vee h \vee b)$$
$$\wedge\ (\neg h \vee \neg g \vee \neg a)$$
$$\wedge\ \neg b$$

Implication Graph:

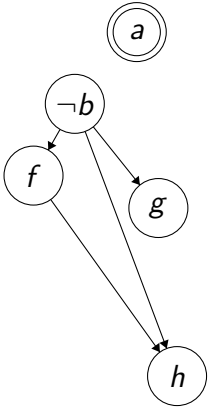


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\wedge\ (a \vee \neg h)$$
$$\wedge\ (\neg b \vee c)$$
$$\wedge\ (\neg b \vee d)$$
$$\wedge\ (\neg c \vee e)$$
$$\wedge\ (\neg e \vee \neg d \vee \neg b)$$
$$\wedge\ (b \vee f)$$
$$\wedge\ (b \vee g)$$
$$\wedge\ (\neg f \vee h \vee b)$$
$$\wedge\ (\neg h \vee \neg g \vee \neg a)$$
$$\wedge\ \neg b$$

Implication Graph:

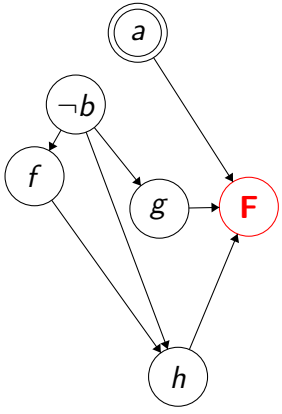


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\wedge\,(a \vee \neg h)$$
$$\wedge\,(\neg b \vee c)$$
$$\wedge\,(\neg b \vee d)$$
$$\wedge\,(\neg c \vee e)$$
$$\wedge\,(\neg e \vee \neg d \vee \neg b)$$
$$\wedge\,(b \vee f)$$
$$\wedge\,(b \vee g)$$
$$\wedge\,(\neg f \vee h \vee b)$$
$$\wedge\,\boxed{(\neg h \vee \neg g \vee \neg a)}$$
$$\wedge\,\neg b$$

Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h) \\
&\wedge (\neg b \vee c) \\
&\wedge (\neg b \vee d) \\
&\wedge (\neg c \vee e) \\
&\wedge (\neg e \vee \neg d \vee \neg b) \\
&\wedge (b \vee f) \\
&\wedge (b \vee g) \\
&\wedge (\neg f \vee h \vee b) \\
&\wedge (\neg h \vee \neg g \vee \neg a) \\
&\wedge \neg b
\end{aligned}
$$

Implication Graph:

CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h) \\
&\wedge (\neg b \vee c) \\
&\wedge (\neg b \vee d) \\
&\wedge (\neg c \vee e) \\
&\wedge (\neg e \vee \neg d \vee \neg b) \\
&\wedge (b \vee f) \\
&\wedge (b \vee g) \\
&\wedge (\neg f \vee h \vee b) \\
&\wedge (\neg h \vee \neg g \vee \neg a) \\
&\wedge \neg b
\end{aligned}
$$

Implication Graph:

CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h)\\
&\wedge (\neg b \vee c)\\
&\wedge (\neg b \vee d)\\
&\wedge (\neg c \vee e)\\
&\wedge (\neg e \vee \neg d \vee \neg b)\\
&\wedge (b \vee f)\\
&\wedge (b \vee g)\\
&\wedge (\neg f \vee h \vee b)\\
&\wedge (\neg h \vee \neg g \vee \neg a)\\
&\wedge \neg b
\end{aligned}
$$

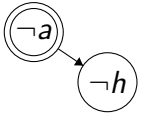
Implication Graph:

CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h) \\
&\wedge (\neg b \vee c) \\
&\wedge (\neg b \vee d) \\
&\wedge (\neg c \vee e) \\
&\wedge (\neg e \vee \neg d \vee \neg b) \\
&\wedge (b \vee f) \\
&\wedge (b \vee g) \\
&\wedge (\neg f \vee h \vee b) \\
&\wedge (\neg h \vee \neg g \vee \neg a) \\
&\wedge \neg b
\end{aligned}
$$

Implication Graph:

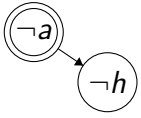


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h) \\
&\wedge (\neg b \vee c) \\
&\wedge (\neg b \vee d) \\
&\wedge (\neg c \vee e) \\
&\wedge (\neg e \vee \neg d \vee \neg b) \\
&\wedge (b \vee f) \\
&\wedge (b \vee g) \\
&\wedge (\neg f \vee h \vee b) \\
&\wedge (\neg h \vee \neg g \vee \neg a) \\
&\wedge \neg b
\end{aligned}
$$

Implication Graph:

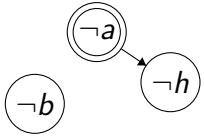


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h) \\
&\wedge (\neg b \vee c) \\
&\wedge (\neg b \vee d) \\
&\wedge (\neg c \vee e) \\
&\wedge (\neg e \vee \neg d \vee \neg b) \\
&\wedge (b \vee f) \\
&\wedge (b \vee g) \\
&\wedge (\neg f \vee h \vee b) \\
&\wedge (\neg h \vee \neg g \vee \neg a) \\
&\wedge \neg b
\end{aligned}
$$

Implication Graph:

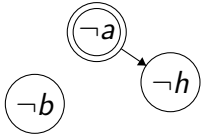


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\wedge\,(a \vee \neg h)$$
$$\wedge\,(\neg b \vee c)$$
$$\wedge\,(\neg b \vee d)$$
$$\wedge\,(\neg c \vee e)$$
$$\wedge\,(\neg e \vee \neg d \vee \neg b)$$
$$\wedge\,(b \vee f)$$
$$\wedge\,(b \vee g)$$
$$\wedge\,(\neg f \vee h \vee b)$$
$$\wedge\,(\neg h \vee \neg g \vee \neg a)$$
$$\wedge\,\neg b$$

Implication Graph:

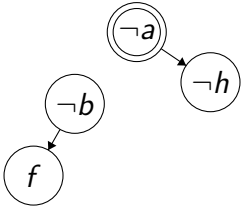


CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$\wedge\ (a \vee \neg h)$$
$$\wedge\ (\neg b \vee c)$$
$$\wedge\ (\neg b \vee d)$$
$$\wedge\ (\neg c \vee e)$$
$$\wedge\ (\neg e \vee \neg d \vee \neg b)$$
$$\wedge\ (b \vee f)$$
$$\wedge\ (b \vee g)$$
$$\wedge\ (\neg f \vee h \vee b)$$
$$\wedge\ (\neg h \vee \neg g \vee \neg a)$$
$$\wedge\ \neg b$$

Implication Graph:



CDCL: Motivating Example Revisited

Looking for satisfying interpretation of:

$$
\begin{aligned}
&\wedge (a \vee \neg h)\\
&\wedge (\neg b \vee c)\\
&\wedge (\neg b \vee d)\\
&\wedge (\neg c \vee e)\\
&\wedge (\neg e \vee \neg d \vee \neg b)\\
&\wedge (b \vee f)\\
&\wedge (b \vee g)\\
&\wedge \boxed{(\neg f \vee h \vee b)}\\
&\wedge (\neg h \vee \neg g \vee \neg a)\\
&\wedge \neg b
\end{aligned}
$$

Implication Graph: