SAT Solving and DPLL

COM 270: Introduction to Automated Deduction

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SAT Solvers in Practice

- SAT solvers are typically available as part of optimization libraries, e.g. Google OR tools (C++, Java, Python, .NET)
- More general-purpose SMT solvers, such as Z3 can also act as SAT solvers.
- Here is an online SAT solver (not efficient but useful for demonstration): `https://www.inf.ufpr.br/dpasqualin/d3-dpll/`

Recall Smullyan's Problem

- "The only possible suspects were the Cook, the Duchess, and the Cheshire Cat.
  "The Cheshire Cat stole it!" said the Duchess at the trial.
  "Oh, yes, I stole it!" said the Cheshire Cat with a grin.
  "I didn't steal it!" said the Cook.
  As it turned out, the thief had lied and at least one of the others had told the truth.
  Who stole the cookbook?"
- Introduce variables:

$D =$ "The Duchess stole the cookbook"	$d =$ "The Duchess told the truth"
$C =$ "The Cheshire Cat stole the cookbook"	$c =$ "The Cheshire Cat told the truth"
$K =$ "The Cook stole the cookbook"	$k =$ "The Cook told the truth"

- In CNF after applying Tseytin's:
  $(\neg d \vee C) \wedge (\neg C \vee d) \wedge (\neg c \vee C) \wedge (\neg C \vee c) \wedge (\neg k \vee \neg K) \wedge (K \vee k) \wedge (\neg D \vee X) \wedge (\neg K \vee Y) \wedge (\neg C \vee Z) \wedge (\neg x \vee k \vee c) \wedge (\neg k \vee x) \wedge (\neg c \vee x) \wedge (\neg y \vee c \vee d) \wedge (\neg c \vee y) \wedge (\neg d \vee y) \wedge (\neg z \vee d \vee k) \wedge (\neg d \vee z) \wedge (\neg k \vee z) \wedge (\neg X \vee \neg d) \wedge (\neg X \vee x) \wedge (d \vee \neg x \vee X) \wedge (\neg Y \vee \neg k) \wedge (\neg Y \vee y) \wedge (k \vee \neg y \vee Y) \wedge (\neg Z \vee \neg c) \wedge (\neg Z \vee z) \wedge (c \vee \neg z \vee Z) \wedge (D \vee C \vee K)$

Smullyan's Solution

- "It is impossible that the Cheshire Cat stole it, because the thief would then be telling the truth. Therefore the Cheshire Cat didn't steal it (and the Cat and the Duchess were both lying). If the Cook stole it..."

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- Smullyan assumes that Cheshire Cat stole the cookbook, propagates the constraints to derive what this assumption implies, then arrives at a conflict, then backtracks.

Backtracking + Boolean Constraint Propagation

Looking for satisfying interpretation of:

$$
\begin{aligned}
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Implication Graph:

Backtracking + Boolean Constraint Propagation

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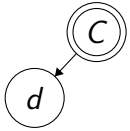

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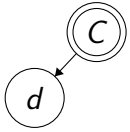

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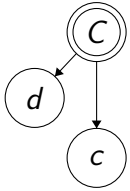

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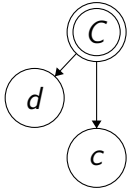

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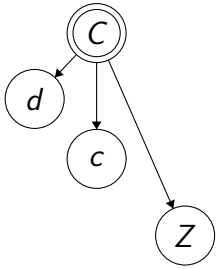

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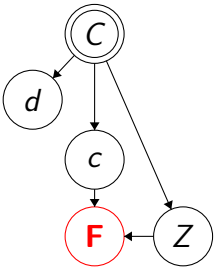

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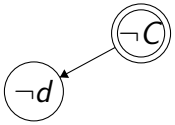

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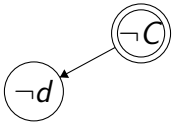

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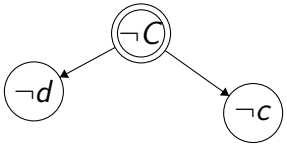

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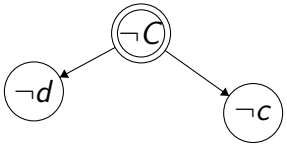

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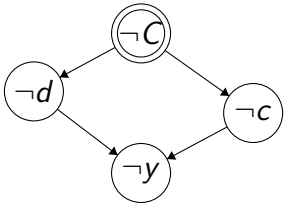

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$\wedge\ (\neg C \vee Z)$

$\wedge\ (\neg x \vee k \vee c) \wedge (\neg k \vee x) \wedge (\neg c \vee x)$

$\wedge\ (\neg y \vee c \vee d) \wedge (\neg c \vee y) \wedge (\neg d \vee y)$

$\wedge\ (\neg z \vee d \vee k) \wedge (\neg d \vee z) \wedge (\neg k \vee z)$

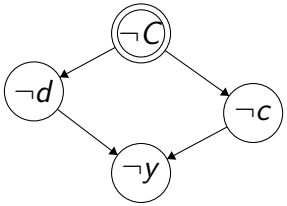
$\wedge\ (\neg X \vee \neg d) \wedge (\neg X \vee x) \wedge (d \vee \neg x \vee X)$

$\wedge\ (\neg Y \vee \neg k) \wedge (\neg Y \vee y) \wedge (k \vee \neg y \vee Y)$

$\wedge\ (\neg Z \vee \neg c) \wedge (\neg Z \vee z) \wedge (c \vee \neg z \vee Z)$

$\wedge\ (D \vee C \vee K)$

Implication Graph:



Backtracking + Boolean Constraint Propagation

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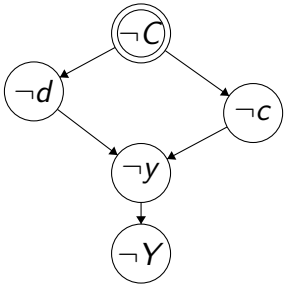

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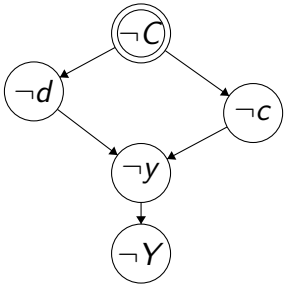

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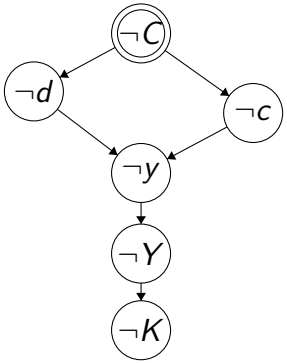

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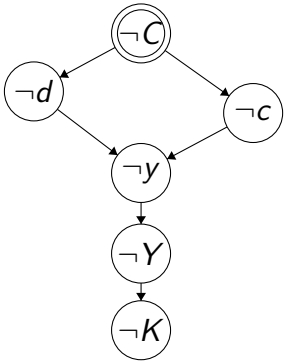

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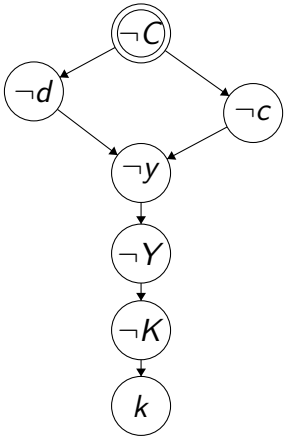

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$$
\begin{aligned}
&\wedge (\neg d \vee C) \wedge (\neg C \vee d) \\
&\wedge (\neg c \vee C) \wedge (\neg C \vee c) \\
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&\wedge (\neg D \vee X) \\
&\wedge (\neg K \vee Y \\
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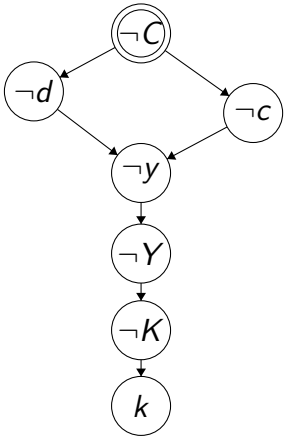

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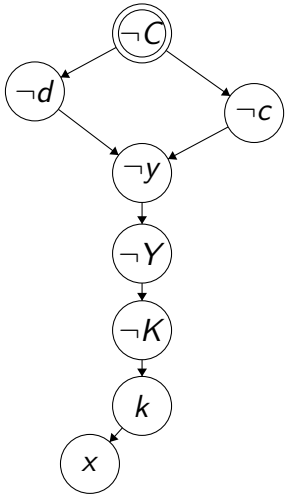

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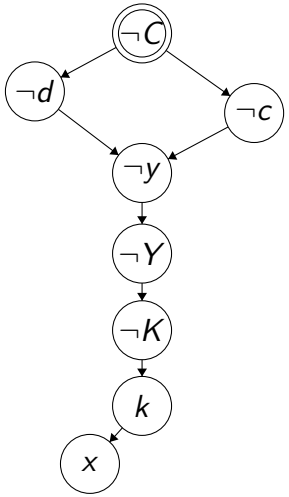

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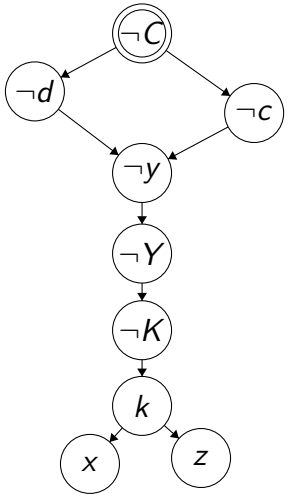

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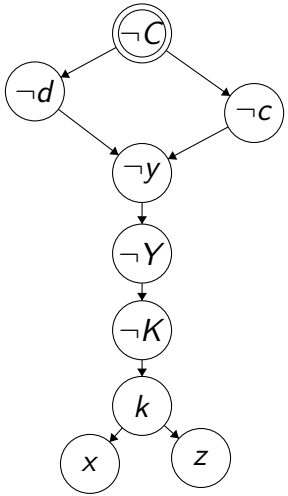

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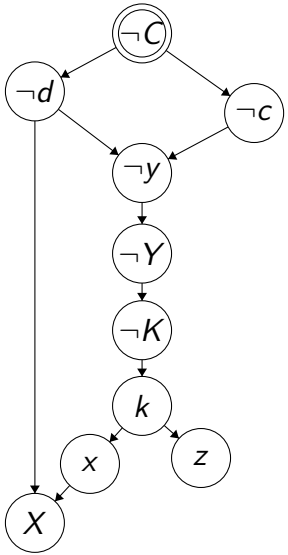

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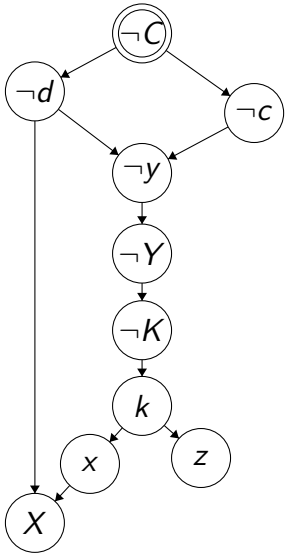

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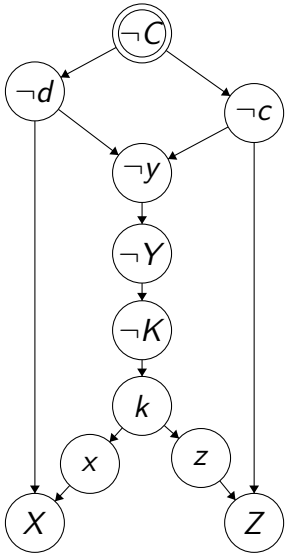

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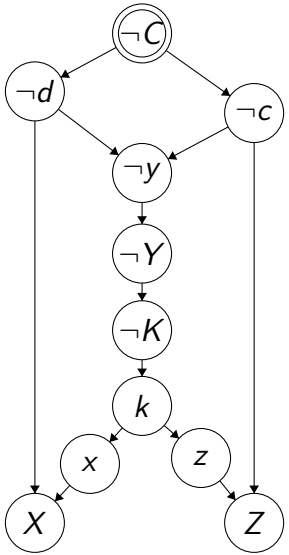

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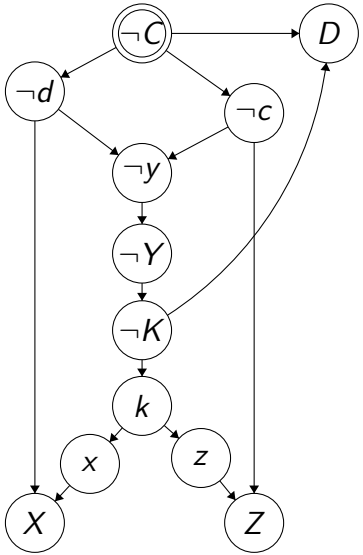

DPLL (DavisPutnamLogemannLoveland) Algorithm

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- DPLL also uses **pure literal elimination** to remove easy-to-satisfy clauses.

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- SAT solvers eliminate all pure literals as the first step in the solving process.

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- SAT solvers aim to find a satisfying interpretation for a given propositional formula.
- Backtracking assumes a variable's value and reverses it if a conflict arises.
- Boolean constraint propagation determines variable assignments implied by current assumptions.
- DPLL combines backtracking with Boolean constraint propagation.