# Probability Basics, Naive Bayes

## COM 214: Introduction to Artificial Intelligence

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Tufts University

# Outline

1. Probability Basics
2. Naive Bayes Classifier
3. An Example NLP Problem

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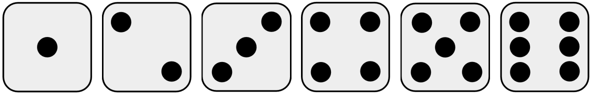

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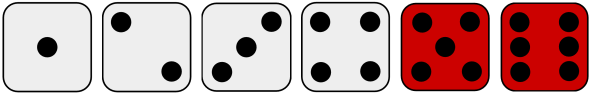

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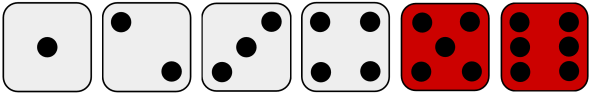

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- Therefore, the probability is $P(>4) = \frac{2}{6} = \frac{1}{3}$.

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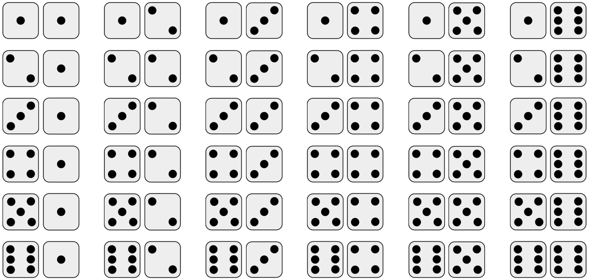

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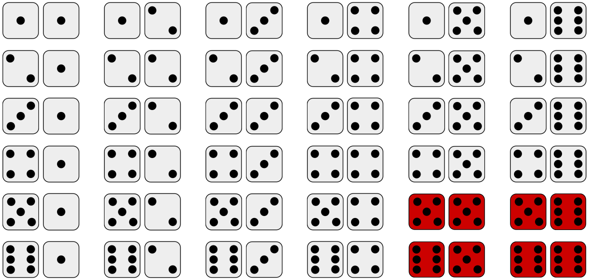

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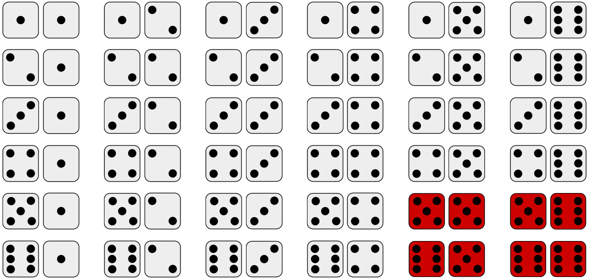

Useful in board games:)

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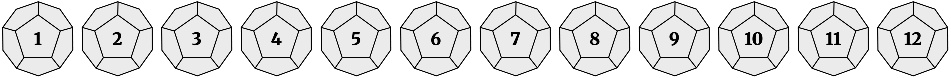

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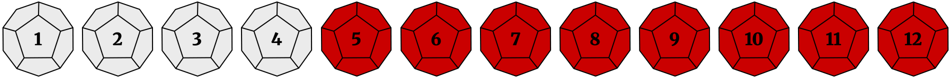


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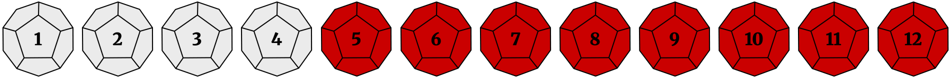


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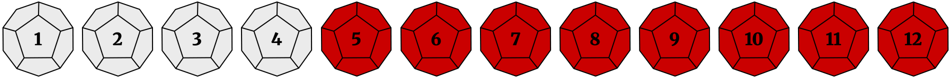


- We get a number greater than 4 in 8 cases.
- Therefore, the probability is $P(>4) = \frac{8}{12} = \frac{2}{3}$.

# Conditional Probability, Bayes Theorem

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- More formally, $P(a \mid b) = \frac{P(a \text{ AND } b)}{P(b)}$.
- From this we get Bayes Theorem:

$$P(a \mid b) = \frac{P(b|a)P(a)}{P(b)}$$

# Thomas Bayes

**Thomas Bayes** (/beɪz/ *BAYZ*, audio; c. 1701 – 7 April 1761[2][4][note 1]) was an English statistician, philosopher and Presbyterian minister who is known for formulating a specific case of the theorem that bears his name: Bayes' theorem.

Bayes never published what would become his most famous accomplishment; his notes were edited and published posthumously by Richard Price.[5]

The Reverend
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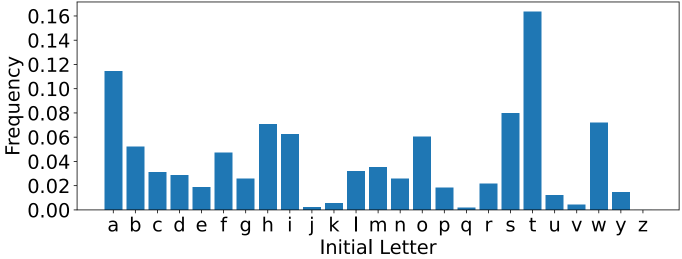

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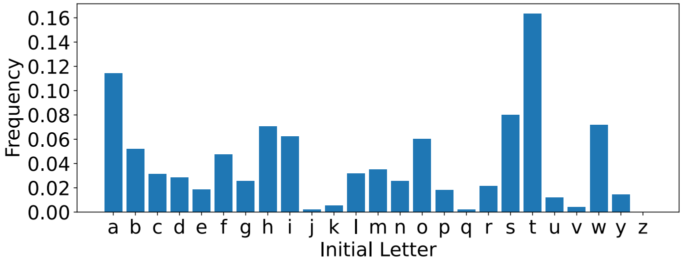


- Then $P(AUCAISGREAT) = P(A)^3 \times P(U) \times P(C) \times P(I) \times P(S) \times P(G) \times P(R) \times P(E) \times P(T)$

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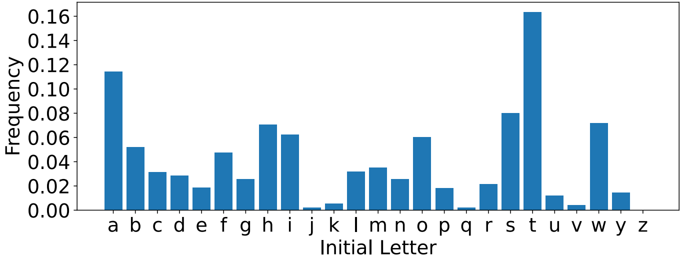


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- This is a naive approach because these probabilities aren't actually independent... Hence *Naive Bayes*

# Classification With Language Models, Ngrams

| Class A: | Class B: |
| --- | --- |
| $P(A \mid data)$ | $P(B \mid text)$ |
| $\frac{P(text \mid A) P(A)}{P(text)}$ | $\frac{P(text \mid B) P(B)}{P(text)}$ |

$P(text|C) = P(w_1|C) * P(w_2|C)...P(w_n|C) \leftarrow$ Naive Bayes

$P(text|C) = P(w_1|C) * P(w_2|w_1, C)...P(w_n|w_{n-1}, C) \leftarrow$ 2-grams

$P(text|C) = P(w_1|C) * P(w_2|w_1, w_2, C)...P(w_n|w_{n-2}, w_{n-1}, C) \leftarrow$ 3-grams

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# Motivation: Acrostics

**Edgar Allan Poe, 1829**

Elizabeth it is in vain you say
"Love not"—thou sayest it in so sweet a way:
In vain those words from thee or L. E. L.
Zantippe's talents had enforced so well:
Ah! if that language from thy heart arise,
Breathe it less gently forth—and veil thine eyes.
Endymion, recollect, when Luna tried
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- Uncovering an acrostic may prove authorship or reveal hidden knowledge.

# Goal: Uncovering New Acrostics

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$= \frac{P(t_1|a)[P(t_2|a)P(a)+P(t_2|\text{not } a)P(\text{not } a)]}{[P(t_1|a)P(a)+P(t_1|\text{not } a)P(\text{not } a)]P(t_2|a)}$

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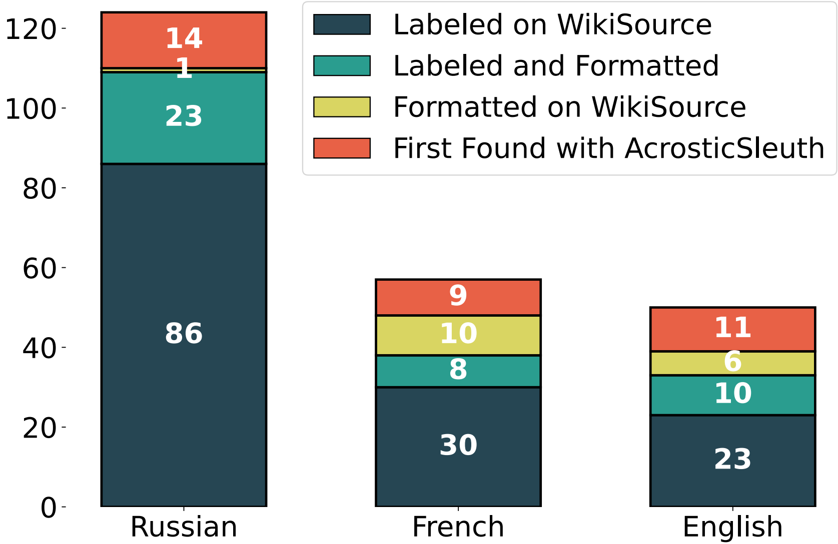
- To solve the problem we only need to know $P(t|a)$ and $P(t|\text{not } a)$.

# AcrosticSleuth Results



# Spotlight: Thomas Hobbes

**Thomas Hobbes** (/hɒbz/ *HOBZ*; 5 April 1588 – 4 December 1679) was an English philosopher, best known for his 1651 book *Leviathan*, in which he expounds an influential formulation of social contract theory.[4] He is considered to be one of the founders of modern political philosophy.[5][6]

In his early life, overshadowed by his father's departure following a fight, he was taken under the care of his wealthy uncle. Hobbes's academic journey began in Westport, leading him to Oxford University

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